

# Consistent analysis of the masses and decays of the $[70, 1^-]$ baryons in the $1/N_c$ expansion

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## PLAN

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- Baryons in the  $1/N_c$  expansion of QCD
- Independent analysis of masses and decays of the  $[70, 1^-]$ -plet
- Consistent analysis of masses and decays of the  $[70, 1^-]$ -plet
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## INTRODUCTION & MOTIVATION

Most of our understanding of the excited baryon sector is largely based on data analyzed by models, most prominently the constituent quark model, which relation with QCD is, at least, unclear.

Although there has been recently important progress in the study of the excited baryons using lattice QCD simulations this remains to be a very hard problem .

In this situation it is important to have a model independent approach to the physics of excited baryons.

One possible systematic approach of this type is the  $1/N_c$  expansion of QCD.

	SU(6) irrep	SU(3) <sub>f</sub> irrep	J <sup>P</sup>	S = 0
<b>GS Baryons</b>	<b>56<sup>+</sup>(I=0)</b>	<b><sup>2</sup>8</b> <b><sup>4</sup>10</b>	<b>1/2<sup>+</sup></b> <b>3/2<sup>+</sup></b>	<b>N(939)</b> <b>Δ(1232)</b>
<b>Excited baryons</b>	<b>70<sup>-</sup>(I=1)</b>	<b><sup>2</sup>8</b>	<b>3/2<sup>-</sup></b>	<b>N(1520)</b>
			<b>1/2<sup>-</sup></b>	<b>N(1535)</b>
			<b>1/2<sup>-</sup></b>	<b>N(1650)</b>
			<b>5/2<sup>-</sup></b>	<b>N(1675)</b>
		<b><sup>2</sup>10</b>	<b>3/2<sup>-</sup></b>	<b>N(1700)</b>
			<b>1/2<sup>-</sup></b>	<b>Δ(1620)</b>
			<b>3/2<sup>-</sup></b>	<b>Δ(1700)</b>
			<b><sup>2</sup>1</b>	<b>1/2<sup>-</sup></b>
		<b>3/2<sup>-</sup></b>		

We can define the mixing angles

$$\begin{pmatrix} N_J \\ N'_J \end{pmatrix} = \begin{pmatrix} \cos \theta_{2J} & \sin \theta_{2J} \\ -\sin \theta_{2J} & \cos \theta_{2J} \end{pmatrix} \begin{pmatrix} {}^2N_J \\ {}^4N_J \end{pmatrix}$$

with  $J = 1/2, 3/2$

“Standard” QM values for these angles are  $\theta_1 = 0.61(0.09)$ ,  $\theta_3 = 3.04(0.15)$  mainly from strong decays analyses. [Numbers in parenthesis are uncertainties.]

Within the  $1/N_c$  expansion an analysis of the baryon masses lead to (Carlson et al, PRD59 (99) 114008)  $\theta_1 = 0.55(0.37)$  ;  $\theta_3 = 3.00(0.21)$

This has to be compared with the result obtained in the  $1/N_c$  strong decay analysis (Goity et al, PRD71 (05) 034016)  $\theta_1 = 0.39(0.11)$  ;  $\theta_3 = [2.38, 2.82](0.11)$ . Higher value of  $\theta_3$  seems to be preferred from e.m.helicity amplitudes (NNS et al. PLB663 (08)222)

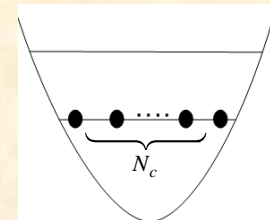
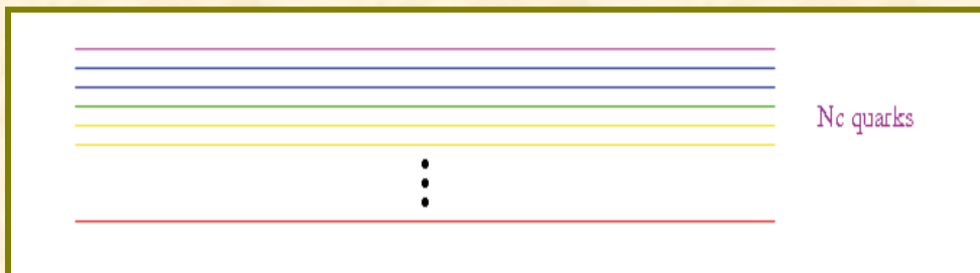
It is thus important to see whether a simultaneous fit of masses and decays is possible.

# BARYONS IN THE $1/N_c$ EXPANSION OF QCD

QCD has no obvious expansion parameter. However, t'Hooft ('74) realized that if one extends the QCD color group from  $SU(3)$  to  $SU(N_c)$ , where  $N_c$  is an arbitrary (odd) large number, then  $1/N_c$  may be treated as the relevant expansion parameter of QCD. To have consistent theory, QCD coupling constant  $g^2 \sim 1/N_c$

*For large  $N_c$  there are infinite mesons states, which are narrow and weakly coupled between each other.* (t'Hooft '74)

Witten ('79) observed that baryons are formed by  $N_c$  "valence" quarks with  $M_B \sim O(N_c^1)$  and  $r_B \sim O(N_c^0)$ . *In the Large  $N_c$  limit a Hartree picture of baryons emerges: each quark moves in a self-consistent effective potential generated by the rest of the  $(N_c-1)$  quarks.*



GS  
Baryon

Moreover, in order to preserve unitarity for Large  $N_c$ , a dynamical spin-flavor arises in the baryon sector [[Gervais-Sakita '84](#); [Dashen-Jenkins-Manohar '93](#)]

$$S^i, T^a, X^{ia} = \frac{G_{ia}}{N_c} \quad \text{form contracted } \text{SU}(2 N_f) \text{ algebra}$$

Here,  $G_{ia}$  spin-flavor operator. E.g. in the quark representation  $G_{ia} = \sum_j q_j^\dagger \sigma_i \tau_a q_j$

To derive a  $1/N_c$  expansion of baryonic observables one can make use of the contracted algebra (“consistency relation method”).

Alternatively, one can use the usual  $\text{SU}(2 N_f)$  algebra for large  $N_c$ . In this scheme (so-called “operator method”) GS baryons are taken to fill  $\text{SU}(2 N_f)$  completely symmetric irrep.

Quark operator method: Any color singlet QCD operator can be represented at the level of effective theory by a series of composite operators ordered in powers of  $1/N_c$

$$O_{eff} = \sum_{n,i} c_i^{(n)} \Phi_i^{(n)}$$

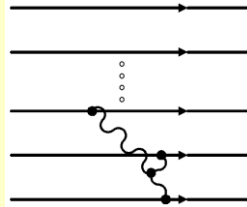
$n$ -body operator obtained as the product of  $n$  generators of  $SU(2 N_f)$ , i.e.  $S^i, T^a, G^{ia}$

Unknown coefficients to be fitted

Rules for  $N_c$  counting

- $n$ -body operators need at least  $n$  quarks exchanging  $(n-1)$  gluons according to usual rules it carries a suppression factor  $(1/N_c)^{n-1}$

e.g. 3-body operator



$$g^4 \sim N_c^{-2}$$

- Some operators may act as coherent  $\rightarrow$  matrix elements  $O(N_c)$  e.g.  $G^{ia} \sim N_c$

Several reduction formulae exist (Dashen, Jenkins, Manohar, PRD49(94)4713; D 51(95), 3697) that allow to reduce the number of relevant operators to be considered

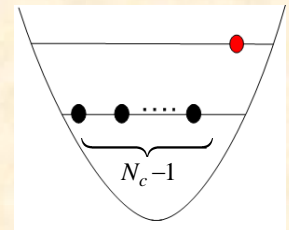
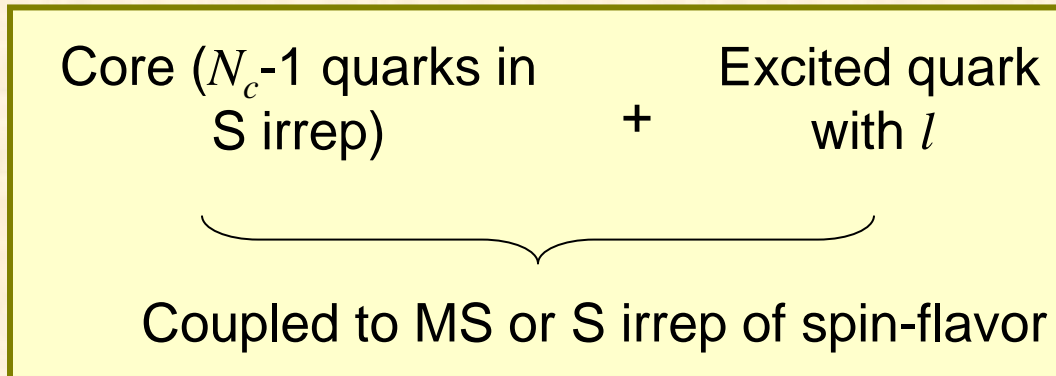
This type of analysis has been applied to study axial couplings, magnetic moments, etc. (Dai, Dashen, Jenkins, Manohar, PRD53(96)273, Carone, Georgi, Osofsky, PLB322(94)227, Luty, March-Russell, NPB426(94)71, etc).



Carone et al, PRD50(94)5793, Goity, PLB414(97)140; Pirjol and Yan, PRD57(98)1449; 5434 proposed to extend these ideas to analyze low lying excited baryons properties.

Take as convenient basis of states: multiplets of  $O(3) \times SU(2 N_f)$  (approximation since they might contain several irreps of  $SU(2N_f)_c$  Schat, Pirjol, PRD67(03)096009, Cohen, Lebed, PLB619(05)115)

Excited baryon composed by



Low lying Excited Baryon

For lowest states relevant multiplets are  $[1^-, 70]$  ,  $[0^+, 56']$  ,  $[2^+, 56]$  ...

In the operator analysis, effective  $n$ -body operators are now

$O^{(n)} = R \otimes \Phi^{(n)}$  where  $R$  is an  $O(3)$  operator and  $\Phi^{(n)}$  an  $SU(2N_f)$  tensor

$$\Phi^{(n)} = \frac{1}{N_c^{n-1}} \lambda \otimes \underbrace{\Lambda_c \otimes \dots \otimes \Lambda_c}_{n-1} \quad \text{where} \quad \begin{cases} \lambda = t^a, s^i, g^{ia} \\ \Lambda_c = T_c^a, S_c^i, G_c^{ia} \end{cases}$$

There is by now quite a number of works on the application of this approach to the analysis of the excited baryons observables. Some of these works are

Excited baryon masses

Carlson, Carone, Goity and Lebed, PLB438 (98) 327; PRD59 (99) 114008.

Schat, Goity and NNS, PRL88(02)102002, PRD66 (02) 114014, PLB564 (03) 83

Matagne, Stancu, PRD71(05)014010, PLB631(05)7, PRD74(06)034014, .....

Pirjol, Schat, PRD67(03)096009; PRD78(08)034026; PRD80(09)116004,.....

Strong decays: Carlson et al, PRD59(99)114008; Goity et al, PRD71(05)034016, PRD80(09)074027, .....

E.M. Helicity amplitudes: Carlson, Carone, PRD58(98)053005; Goity, NNS, PRL99 (07) 062002; NNS, Goity, Matagne, PLB663(08)222.....



# INDEPENDENT ANALYSIS OF MASSES AND DECAYS

Analysis of the masses non-strange (1-,70) resonances

Carlson, Carone, Goity and Lebed, PLB438 (98) 327; PRD59 (99) 114008,...

Masses given by diagonal m.e. of mass operator M except for j=1/2,3/2 nucleon states where one has to diagonalize 2 x 2 matrices. In those cases

$$m_{N_J, N'_J} = \frac{M_2^J + M_4^J}{2} \mp \sqrt{\left(\frac{M_2^J + M_4^J}{2}\right)^2 + (M_{24}^J)^2}$$

$$\tan 2\theta_{2J} = \frac{2M_{24}^J}{M_2^J - M_4^J}$$

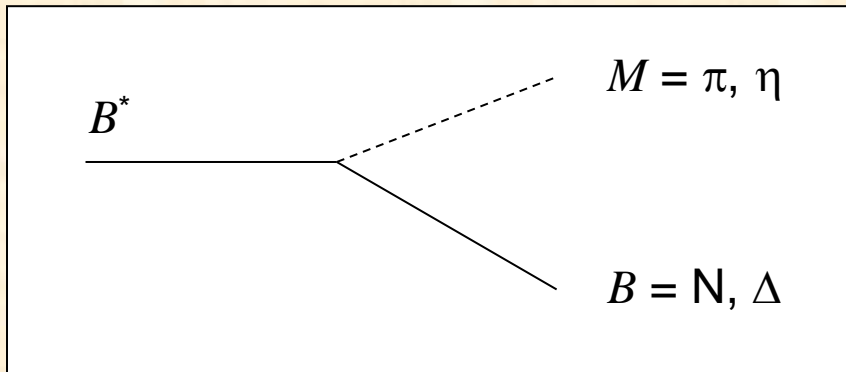
where

$$M_{24}^J = \langle {}^2N_J | M | {}^4N_J \rangle, \text{ etc}$$

$O(1/N_c)$	Operator	Coefficient
$N_c$	$N_c \mathbf{I}$	$498 \pm 9$
1	$\frac{6}{5} \sqrt{6} (ls)^{[0,0]}$	$23 \pm 30$
	$\frac{144\sqrt{6}}{5N_c} \left( (ll)^{(2)} (gG_c)^{[2,0]} \right)^{[0,0]}$	$-37 \pm 4$
	$\sqrt{\frac{8}{3}} \left( -ls + \frac{12}{N_c+3} l (tG_c)^{[1,0]} \right)^{[0,0]}$	$62 \pm 129$
$1/N_c$	$\frac{9}{5} \frac{1}{N_c} (lS_c)^{[0,0]}$	$-102 \pm 78$
	$-\frac{9}{2\sqrt{2}} \frac{1}{N_c} (S_c S_c)^{[0,0]}$	$-544 \pm 124$
	$\frac{3\sqrt{3}}{N_c} (sS_c)^{[0,0]}$	—
	$\frac{6\sqrt{6}}{N_c} \left( (ll)^{(2)} (sS_c)^{[2,0]} \right)^{[0,0]}$	—
$\theta_1$		$0.55 \pm 0.37$
$\theta_3$		$3.00 \pm 0.21$
$\chi_{dof}^2$		0.26

# Analysis of strong decays of non-strange ( $1^-, 7_0$ ) resonances

We consider decays of the type



where  $B^*$  are non-strange members of the  $[1^-, 7_0]$ . Relevant partial waves are  $l_M = S, D$

The decay widths are given by

$$\Gamma^{[l_M, i_M]} = \frac{k_M^{2l_M+1}}{8\pi^2 \Lambda^{2l_M}} \frac{M_B}{M_{B^*}} \frac{\left| \sum_n C_n^{[l_M, i_M]} \langle J, I || (\mathcal{B}^{[l_M, i_M]})_n || J^*, I^*, S^* \rangle \right|^2}{(2I^* + 1)(2J^* + 1)}$$

where

$$\mathcal{B}^{[l_M, i_M]} = \left( \xi^{(l)} \Phi^{[l, i_M]} \right)^{[l_M, i_M]}$$

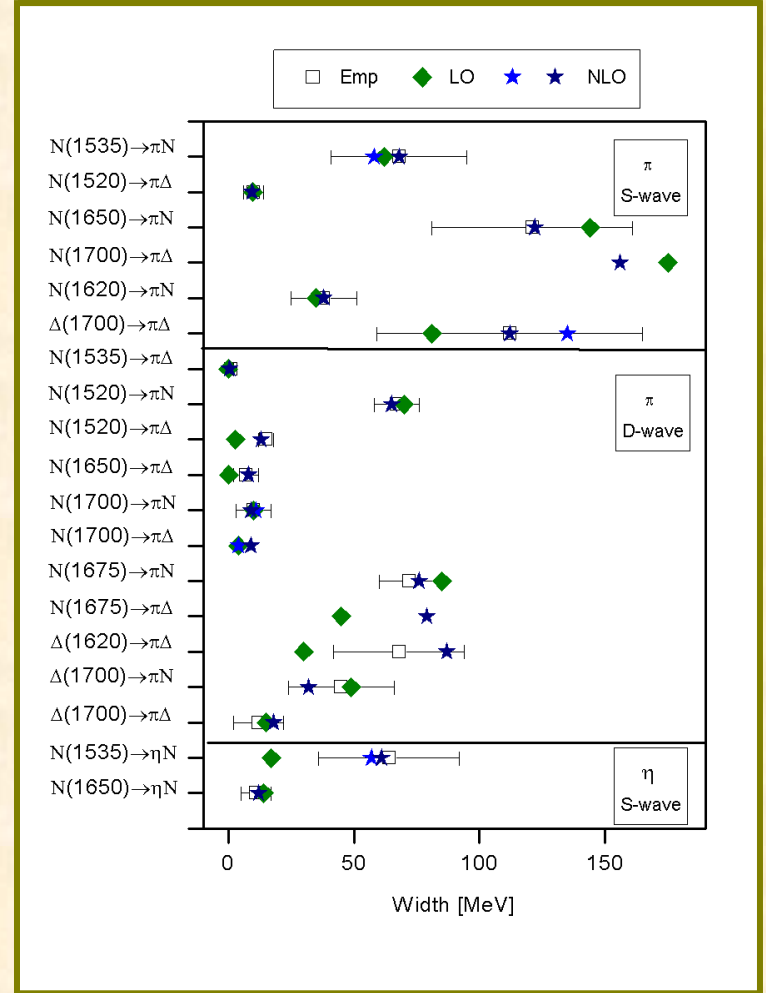
acts orbital wf of excited quark

acts of spin-flavor wf of the excited quark – core system

# Goity, Schat and NNS, PRD71(05)034016

Basis operators and fit parameters for the decay of non-strange baryons of the  $(1^-, \mathbf{70})$ -plet. Errors in parenthesis. Square brackets imply that two solutions with very similar  $\chi^2$  exist.

Operator		LO	NLO
$(\xi g)^{[0,1]}$		31(3)	23(3)
Pion	$\frac{1}{N_c} \left( \xi (s T_c)^{[1,1]} \right)^{[0,1]}$	-	[7, 32]([30, 40])
S wave	$\frac{1}{N_c} \left( \xi (t S_c)^{[1,1]} \right)^{[0,1]}$	-	[21, 27](15)
	$\frac{1}{N_c} \left( \xi (g S_c)^{[1,1]} \right)^{[0,1]}$	-	[-26, -67]([40, 65])
$(\xi g)^{[2,1]}$		4.6(0.5)	3.4(0.3)
Pion	$\frac{1}{N_c} \left( \xi (s T_c)^{[1,1]} \right)^{[2,1]}$	-	-4.5(2.4)
	$\frac{1}{N_c} \left( \xi (t S_c)^{[1,1]} \right)^{[2,1]}$	-	[-0.01, 0.08](2)
D wave	$\frac{1}{N_c} \left( \xi (g S_c)^{[1,1]} \right)^{[2,1]}$	-	5.7(4)
	$\frac{1}{N_c} \left( \xi (g S_c)^{[2,1]} \right)^{[2,1]}$	-	3.0(2.2)
	$\frac{1}{N_c} \left( \xi (s G_c)^{[2,1]} \right)^{[2,1]}$	[-1.86, -2.25](0.4)	-1.73(0.26)
$(\xi s)^{[0,0]}$		11(4)	17(4)
S wave	$\frac{1}{N_c} \left( \xi (s S_c)^{[1,0]} \right)^{[0,0]}$	-	-
$\theta_1$		1.56(0.15) 0.35(0.14)	0.39(0.11)
$\theta_3$		[3.00, 2.44](0.07)	[2.82, 2.38](0.11)
$\chi_{\text{dof}}^2$		1.5	0.9
dof		10	3



- Dominance of 1B LO operator  $g^{ia}$  in  $\pi$ -decays as in  $\chi$ QM
- Several NLO coefficients poorly determined due to large data error bars. Better emp. inputs needed to determine significance of NLO corrections more precisely.
- $N^*(1535)$  ratio of decays to  $N\eta / N\pi$  well reproduced.

# Helicity amplitudes for $[1^-, 70]$ -plet non-strange resonances

The helicity amplitudes of interest are defined as

$$A_\lambda = -\sqrt{\frac{2\pi\alpha}{\omega_\gamma}} \eta(B^*) \langle B^*, \lambda | \hat{e}_{+1} \cdot \vec{J}(\omega_\gamma \hat{z}) | N, \lambda - 1 \rangle$$

- $\lambda = 1/2, 3/2$  is helicity along  $\gamma$ -momentum (z-axis)
- $\hat{e}_{+1}$  is  $\gamma$ -polarization vector

$\eta(B^*)$  sign factor which depends on sign of strong amplitude  $\pi N \rightarrow B^*$ . When  $B^*$  can decay through 2 partial waves (e.g. S or D)  $\rightarrow$  undetermined sign ( $\xi = S/D = \pm 1$ )

$\vec{J}(\omega_\gamma \hat{z})$  can be represented in terms of effective multipole baryonic operators with isospin  $I = 0, 1$ . Then, the electric and magnetic components of the helicity amplitudes can be expressed as

$$A_\lambda^{ML} = \eta(B^*) \sqrt{\frac{3\alpha N_c}{4\omega_\gamma}} \left(\frac{\omega_\gamma}{m_\rho}\right)^L \sum_{n,I} g_n^{[L,I]} \langle B^*; [\lambda, I_3] | (\mathcal{B}_n)_{[10]}^{[LI]} | N; [\lambda - 1, I_3] \rangle$$

$$A_\lambda^{EL} = \eta(B^*) \sqrt{\frac{3(L+1)\alpha N_c}{4(2L+1)\omega_\gamma}} \left(\frac{\omega_\gamma}{m_\rho}\right)^{L-1} \sum_{n,I} g_n^{[L,I]} \langle B^*; [\lambda, I_3] | (\mathcal{B}_n)_{[10]}^{[LI]} | N; [\lambda - 1, I_3] \rangle$$

where

$$\mathcal{B}^{[L,I]} = \left( \xi^{(I)} \Phi^{[I,I]} \right)^{[L,I]}$$

Acts on orbital wf of excited q

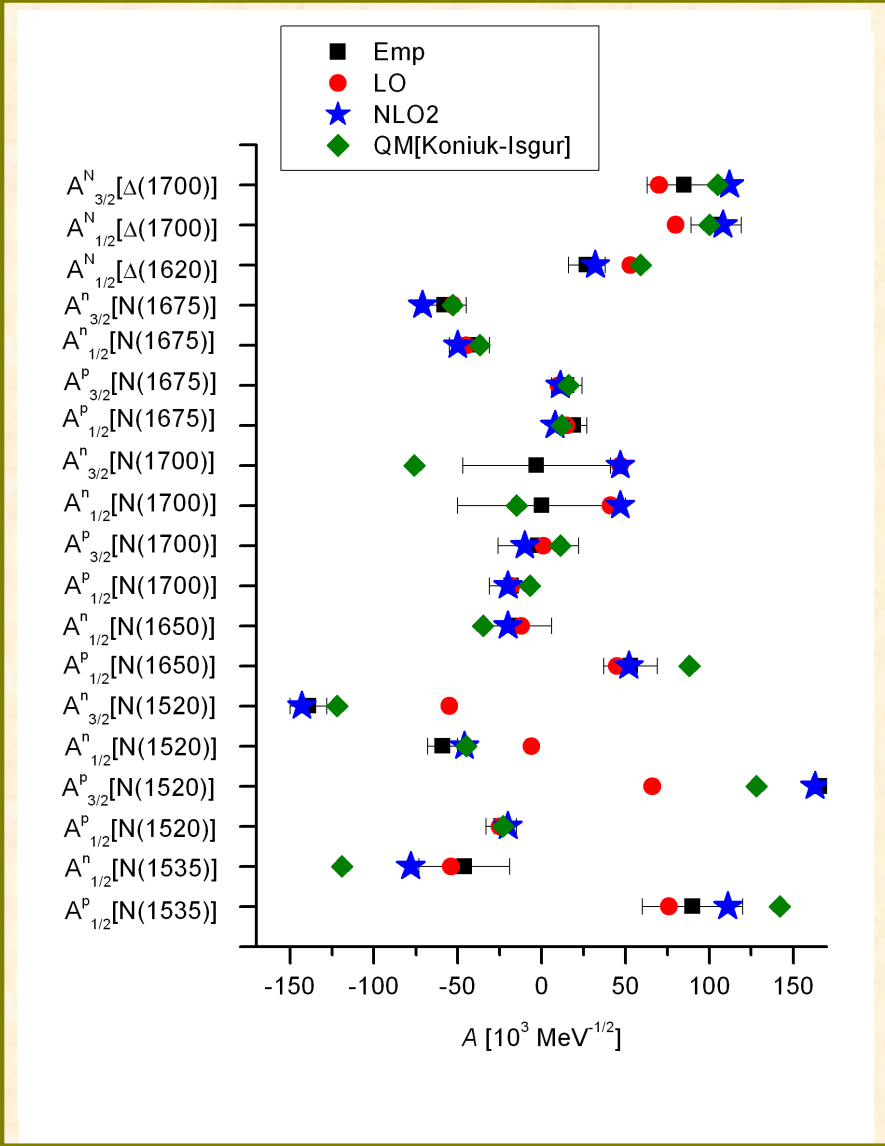
Acts on spin-flavor wf of the excited quark – core system

$\xi = -1$  and  $\theta_3=2.82$  clearly favored by fits. Only this case is shown.

Basis operators and fit parameters of non-strange  $[1^-, 70]$  baryons.  
Errors are indicated in parenthesis.

Operator	LO	NLO1	NLO2
$E1_1^S = (\xi^{[1,0]} s)^{[1,0]}$	-0.4(0.2)	-0.3(0.2)	-0.3(0.2)
$E1_2^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[0,0]})^{[1,0]}$		0.5(0.6)	
$E1_3^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[1,0]})^{[1,0]}$		1.0(0.9)	
$E1_4^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[2,0]})^{[1,0]}$		0.5(0.6)	
$E1_1^V = (\xi^{[1,0]} t)^{[1,1]}$	2.3(0.3)	3.0(0.2)	3.5(0.1)
$E1_2^V = (\xi^{[1,0]} g)^{[1,1]}$	-0.7(0.4)	0.4(0.3)	
$E1_3^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[2,1]})^{[1,1]}$	0.4(0.5)	-0.2(0.4)	
$E1_4^V = \frac{1}{N_c} (\xi^{[1,0]} (s T_c)^{[1,1]})^{[1,1]}$		-1.9(1.4)	
$E1_5^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[0,1]})^{[1,1]}$ $+ \frac{1}{4\sqrt{3}} E1_1^V$		-0.2(0.9)	
$E1_6^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[1,1]})^{[1,1]}$ $+ \frac{1}{2\sqrt{2}} E1_2^V$		4.2(0.9)	3.9(0.8)
$M2_1^S = (\xi^{[1,0]} s)^{[2,0]}$	0.8(0.2)	1.5(0.3)	1.3(0.2)
$M2_2^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[1,0]})^{[2,0]}$		-1.2(1.3)	
$M2_3^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[2,0]})^{[2,0]}$		-1.2(1.7)	
$M2_1^V = (\xi^{[1,0]} g)^{[2,1]}$	3.0(0.6)	3.8(0.6)	3.9(0.4)
$M2_2^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[2,1]})^{[2,1]}$	-3.1(1.0)	-2.3(1.1)	-2.7(0.6)
$M2_3^V = \frac{1}{N_c} (\xi^{[1,0]} (s T_c)^{[1,1]})^{[2,1]}$		-0.1(1.1)	
$M2_4^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[1,1]})^{[2,1]}$ $+ \frac{1}{2\sqrt{2}} M2_1^V$		-1.5(2.4)	
$E3_1^S = \frac{1}{N_c} (\xi^{[1,0]} (s S_c)^{[2,0]})^{[3,0]}$		0.3(0.8)	
$E3_1^V = \frac{1}{N_c} (\xi^{[1,0]} (s G_c)^{[2,1]})^{[3,1]}$	0.7(0.9)	0.3(0.5)	
$\chi_{dof}^2$	2.42	-	0.94
dof	11	0	13

2B



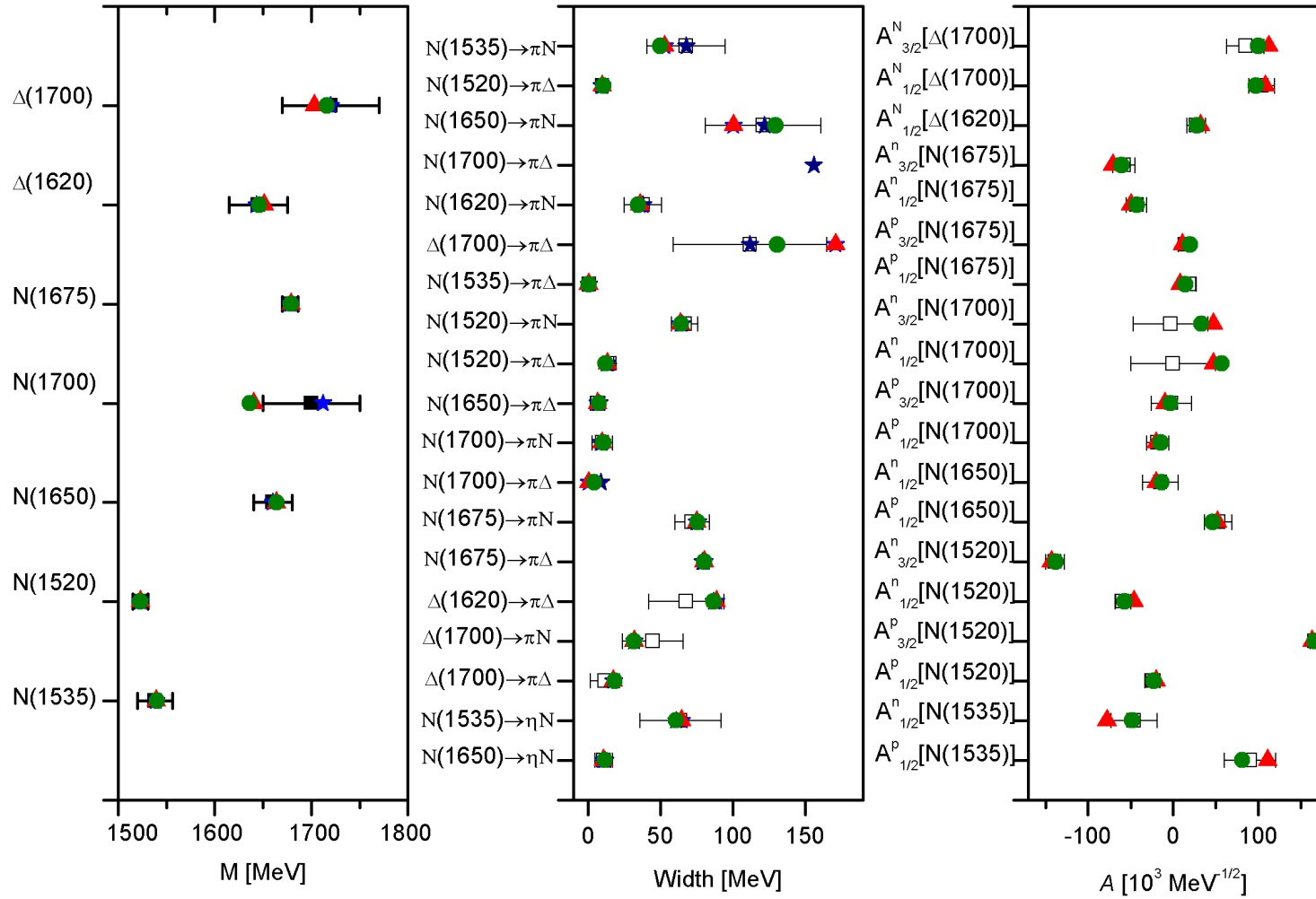
# SIMULTANEOUS ANALYSIS OF MASSES AND DECAYS OF [1-, 70] – PLET BARYONS

To perform consistent analysis: given a set of values for mass coeff.  $C_i$  we determine masses and mixing angles. With those mixing angles and a given set of strong decay coeff.  $C_i^{[l,i]}$  we determine strong decay widths and “strong signs”. With these “strong signs” and mixing angles together with a set of helicity amplitudes coeff.  $ML_i^I$  we determine the helicity amplitudes. We repeat the procedure varying the coefficients  $C_i$ ,  $C_i^{[l,i]}$  and  $ML_i^I$  until a minimum of  $\chi^2_{\text{dof}}$  is found.

		$\theta_1$	$\theta_2$	$\chi^2_{\text{dof}}$
Independent analysis	Masses	0.55(0.37)	3.00(0.21)	0.26
	Strong decays	0.37(0.11)	2.79(0.08)	0.55
				2.36(0.08)
	E.M. Helicity Amplitudes	0.37 (input)	2.79 (input)	0.94
Consistent analysis	Masses & Strong decays	0.42(0.08)	2.75 (0.09)	0.69
	Masses, Strong decays & e.m. helicity amplitudes	0.40(0.08)	2.81(0.10)	0.47
QM		0.61(0.09)	3.04(0.15)	



★ Indep. Analysis    ▲ Mass & Str. decays    ● Mass., Str. decays & E.m. Helicity Amp.



## SUMMARY & CONCLUSIONS

- The operator method for carrying the  $1/N_c$  expansion has been shown to work for GS baryons and it seems to also work for excited baryons. The analyses of masses show small  $O(N_c^0)$  breaking of spin-flavor symmetry. This is dominated by the subleading hyperfine interaction.
- For strong decays, in general, dominance of 1B LO operators. In some cases, as e.g. D wave decays of negative parity excited baryons,  $1/N_c$  corrections not well established due to uncertainties in empirical partial widths.
- In the case of photoproduction amplitudes only a reduced number of the operators in the NLO basis turns out to be relevant. Some of these operators can be identified with those used in QM calculations. However, there are also 2B operators (not included in QM calculations) which are needed for an accurate description of the empirical helicity amplitudes.
- A simultaneous analysis of masses and strong decays of  $[1^-, 70]$  –plet baryons is possible, reducing uncertainties in mixing angles and removing existing ambiguities in independent analysis of masses and strong decays.

Masses							
	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$
I	498(9)	23(30)	-37(4)	62(129)	-102(78)	-544(124)	–
II	512(5)	-16(10)	-11(6)	190(26)	-155(43)	-347(61)	3(2)
III	510(5)	-7(10)	-12(6)	198(26)	-184(44)	-373(62)	4(2)

Strong decays									
	$C_1^{[S;\pi]}$	$C_3^{[S;\pi]}$	$C_4^{[S;\pi]}$	$C_1^{[D;\pi]}$	$C_2^{[D;\pi]}$	$C_4^{[D;\pi]}$	$C_5^{[D;\pi]}$	$C_6^{[D;\pi]}$	$C_1^{[S;\eta]}$
I	23(3)	18(9)	-16(12)	3.4(0.2)	-5(2)	6(3)	3(2)	-1.8(0.2)	17(4)
II	23(3)	18(8)	-	3.3(0.2)	-5(2)	7(3)	3(2)	-1.8(0.2)	18(3)
III	23(3)	17(8)	-17(13)	3.4(0.2)	-5(2)	6(3)	3(2)	-1.7(0.2)	18(3)

E.m. helicity amplitudes								
	$E1_1^S$	$E1_1^V$	$E1_2^V$	$E1_6^V$	$M2_1^S$	$M2_2^S$	$M2_1^V$	$M2_1^V$
I	-0.34 (0.15)	3.5(0.1)	–	3.9(0.8)	1.3(0.2)	–	3.9(0.4)	-2.7(0.6)
II								
III	-0.35 (0.15)	3.2(0.1)	0.46(0.2)	4.1(0.8)	1.6(0.2)	-1.9 (0.5)	3.6(0.5)	-3.0(0.7)

I : Independent analysis

II: Consist. masses and strong decays

III: Consist. Masses, strong decays & e.m. helicity amplitudes